## System of three nonlinear equations solved with the Newton-Raphson method

Solution within the spreadsheet



System of equations f1, f2 and f3
from sheet "1. Solution"

Equation for point A

$$p^2 + 2 \cdot w \cdot p + q^2 - r^2 + w^2 = 0$$

Equation for point B

$$q^2 - r^2 + \frac{w^2}{4} + \frac{w \cdot x}{2} + \frac{x^2}{4} = 0$$

Equation for point D

$$p^2 + q^2 - r^2 + z^2 + 2 \cdot z \cdot q = 0$$

Constants

w = 2
x = 6
z = 3

A matrix "f" is defined as

f = | f1 ; f2 ; f3 |

f1 = p^2 + 2*w*p + q' = 0
f2 = q^2 - r^2 + ((1/4)*w^2 ) + (1/2)*w*x +(1/4)*x^2) = 0
f3 = p^2 + q^2 - r^2 + (z^2 + 2*z*q ) = 0

f = | p^2 + 2*w*p + q^2 - r^2 + w^2 ; q^2-r^2+((1/4)*w^2 + (1/2)*w*x +(1/4)*x^2)) ; p^2 + q^2 - r^2 + (z^2 + 2*z*q ) |



The elements of the matrix "Old", containing the solutions of the system, are initially assumed.

Old = | $p_{assumed}$ ; $q_{assumed}$ ; $r_{assumed}$ |

After each calculation, these elements are replaced by the new obtained values. The iteration ends when the value of all functions "f1"," f2" and "f3" have a value close enough to zero.

**1.- Start of calculation**

The matrix "Old" will contain initially the assumed values and then the values obtained by the iteration

| | | |
|---|---|---|
| **Old =** | 1.000 | ← p |
| | 0.500 | ← q |
| | 4.031 | ← r |

**2.- Matrix "f" evaluated**

The functions f1, f2 and f3 of the matrix "f" are evaluated usung the actual values of the solutions.

**f =** | -7.001 ; -0.001 ; -3.000 |

**3.- Derivatives of the functions**

The derivatives of the functions are:

df1/dp = 2*p + 2*w
df1/dq = 2*q
df1/dr = -2*r
df2/dp = 0
df2/dq = 2*q
df2/dr = -2*r
df3/dp = 2*p
df3/dq = 2*q + 2*z
df3/dr = - 2*r

**4.- Evaluation of the derivatives:**

df1/dp = 6.00
df1/dq = 1.00
df1/dr = -8.06
df2/dp = 0.00
df2/dq = 1.00
df2/dr = -8.06
df3/dp = 2.00
df3/dq = 7.00
df3/dr = -8.06

**5.- Jacobian of the system**

J =

| | | |
|---|---|---|
| df1/dp | df1/dq | df1/dr |
| df2/dp | df2/dq | df2/dr |
| df3/dp | df3/dq | df3/dr |

**6.- Jacobian evaluated**

J =

| | | |
|---|---|---|
| 6.000 | 1.000 | -8.062 |
| 0.000 | 0.000 | -8.062 |
| 2.000 | 7.000 | -8.062 |

**7.- Inverse of Jacobian**

Jinv =

| | | |
|---|---|---|
| 0.175 | -0.150 | -0.025 |
| -0.050 | -0.100 | 0.150 |
| 0.000 | -0.124 | 0.000 |

**8.- Matrix product of Jinv and f**

Jinv * f = (3x3) * (3 x 1)

| | | | | |
|---|---|---|---|---|
| 0.175 | -0.15 | -0.025 | * | -7.001 |
| -0.05 | -0.1 | 0.15 | | -8E-04 |
| 0.0000 | -0.1240 | 0.0000 | | -3 |

Jinv*f = | -1.15 ; -0.0999 ; 1E-04 |

**9.- New solution matrix**

New = Old - Jinv*f

Old = | 1 ; 0.5001 ; 4.0312 | - | -1.15 ; -0.0999 ; 1E-04 |

New = | 2.15 ; 0.6 ; 4.0311 |

Solutions:
Radius r = 4.03114

Iteration

Change the values of matrix "Old" by the values of matrix" New" until the difference between the corresponding elements of both matrices are small enough.

Differences between the "old" and the "new" elements:

Xnew - Xold = 1.2E+00
Ynew - Yold = 1.0E-01
Ynew - Yold = 9.6E-05

Press the iteration macro until the differences are small enough

Iteration macro



## Macro Newton-Raphson

```
Sub Newton_Raphson()
'
' Newton_Raphson Macro
'
' Keyboard Shortcut: Ctrl+r
'
    Range("x38:x40").Select
    Selection.Copy
    Range("e44:e46").Select
    Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
        :=False, Transpose:=False
End Sub
```