

Ecuación a resolver

$$6^x + 4^x = 9^x$$

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$$\frac{6^x}{4^x} + 1 = \frac{9^x}{4^x}$$

$$\frac{3^x \cdot 2^x}{2^x \cdot 2^x} + 1 = \frac{3^x \cdot 3^x}{2^x \cdot 2^x}$$

$$\frac{3^x}{2^x} + 1 = \frac{3^x \cdot 3^x}{2^x \cdot 2^x}$$

$$\left(\frac{3}{2}\right)^x + 1 = \left(\frac{3}{2}\right)^{2x}$$

$$\left(\frac{3}{2}\right)^{2x} - \left(\frac{3}{2}\right)^x - 1 = 0$$

$\frac{\sqrt{5}}{2}$

$$\left(\frac{3}{2}\right)^{2x} - \left(\frac{3}{2}\right)^x - 1 = 0$$

$$\text{sea: } z = \left(\frac{3}{2}\right)^x$$

$$z^2 - z - 1 = 0$$

$$a = 1, b = -1, c = -1$$

$$z = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

$$z = \frac{1 \pm \sqrt{1 + 4}}{2}$$

$$z = \frac{1 \pm \sqrt{5}}{2}$$

$$z_1 = \frac{1 + \sqrt{5}}{2}$$

$$z_1 = \frac{1 - \sqrt{5}}{2}$$

$$z = \left(\frac{3}{2}\right)^x$$

$$z_1 = \left(\frac{3}{2}\right)^{x_1}$$

$$\ln(z_1) = x_1 \cdot \ln\left(\frac{3}{2}\right)$$

$$x_1 = \frac{\ln(z_1)}{\ln\left(\frac{3}{2}\right)}$$

$$x_1 = \frac{\ln\left(\frac{1 + \sqrt{5}}{2}\right)}{\ln\left(\frac{3}{2}\right)}$$

$\frac{\sqrt{5}}{2}$

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similarmente

$$x_2 = \frac{\ln\left(\frac{1 - \sqrt{5}}{2}\right)}{\ln\left(\frac{3}{2}\right)}$$

Esta solución no es válida debido a que contiene el logaritmo de un argumento negativo.

$\frac{\sqrt{5}}{2}$

Solución

$$x_1 = \frac{\ln((1 + \sqrt{5})/2)}{\ln(3/2)}$$

$$x_1 = 1.1868$$

Comprobación

$$6^x = 8.3853$$

$$9^x = 13.568$$

$$+ 4^x = 5.1824$$

$$6^x + 4^x = 13.568$$